# Modelling Particle Entrainment and Particle-Flow Interaction in the Atmospheric Surface Layer

Yaping Shao and An Li
Centre for Advanced Numerical Computation in Engineering and Science,
University of New South Wales, Sydney, Australia

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### Abstract

A numerical model is used to study the hopping motion of sand grains (saltation) in a logarithmic wind. Saltation is considered as a self-limiting process governed by the interaction of four components: aerodynamic entrainment, particle trajectory, splash entrainment and wind modification. In this study, the model is applied to study the modification of surface wind and the increase of surface roughness caused by saltating particles. The numerical results are in qualitative agreement with the previous theoretical analysis.

## 1 Introduction

Saltation is the hopping motion of sand grains along a mobile aeolian surface. It is the key process in wind erosion, because it is not only responsible for sand drift, but also the major mechanism responsible for the entrainment of dust into the atmosphere (Shao et al. 1993). The saltation process involves particle-flow and particle-surface interactions in the atmospheric surface layer. The aerodynamics of the process is a challenging problem that is poorly understood.

Bagnold (1941) assumed that aerodynamic entrainment is the major mechanism responsible for sand drift and developed a simple model for streamwise sand drift. He found that the vertically integrated streamwise sand flux, Q, is proportional to  $u_*^3$  ( $u_*$  is the friction velocity). Owen (1964) considered saltation to be a self-limiting process with three inter-active components: aerodynamic entrainment, particle motion, and modification of wind profile caused by particle momentum transfer.

As a saltating particle impacts the surface, it may rebound and eject more particles into the atmosphere, a process referred to as splash. It is now realized that splash plays an important role in saltation. Anderson and Haff (1991) and McEwan and Willetts (1991) considered saltation as a self-limiting process with four interacting components. In these studies, splash is taken into account in addition to those factors already considered by Owen (1964). Because of the model complexities, solutions are found through numerical integrations.

In this study, we present a new numerical model for the self-limiting saltation process. The model includes three major components: an Eulerian flow model using the k- $\epsilon$  closure; a Lagranian model for particle motion and a parameterization of particle surface interactions. The model is formulated in the spatial domain and consideration is given to the energetic constraints for the choice of some model parameters. The modelling emphasis is places on two particular aspects of the problem: the

modification of wind profile by particle motion and the behavior of saltation depending on the choice of the aerodynamic lift-off coefficient and the splash coefficient.

#### The Numerical Model 2

We consider the saltation of uniform sand grains in a three dimensional coordinate with x aligned with the mean wind, y in the lateral and z in the vertical direction. Saltation is modeled as a selflimiting process determined by four interactive components: (1) modification of wind profile due to particle momentum transfer; (2) particle entrainment by aerodynamic forces; (3) particle trajectory; and (4) particle rebound and entrainment by splash.

#### **Eulerian Flow Model** 2.1

We model turbulent flows in the atmospheric surface layer using the following equation system

$$\frac{\partial u_{f,i}}{\partial x_i} = 0 \tag{1}$$

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$$(1 - \epsilon)\left(\frac{\partial u_{f,i}}{\partial t} + u_{f,j}\frac{\partial u_{f,i}}{\partial x_j}\right) = -(1 - \epsilon)\left(\frac{1}{\rho_f}\frac{\partial p}{\partial x_i} - \frac{1}{R_{ef}}\frac{\partial^2 u_{f,i}}{\partial^2 x_j}\right) - S_i$$
(2)

where  $\epsilon$  is the particle volume fraction in an unit volume.  $S_i$  is the momentum sink due to particle motion, which can be estimated using

$$S_i = \sum \frac{\rho_p \pi d^3}{6} \frac{du_{p,i}}{dt} \tag{3}$$

where the summation is applied to all particles in a unit volume,  $\rho_p$  is particle density, d is particle size, and  $u_{p,i}$  is the ith component of particle velocity. As the particle trajectory is determined using the Lagrangian approach, the momentum sink term can be readily estimated. In the present study, the 3-dimensional flow field is solved using the finite volume method with a conventional k- $\epsilon$  closure.

#### Particle Motion 2.2

The particle trajectory is described by

$$\frac{du_{p,i}}{dt} = \frac{-3}{4} \frac{C_d(R_p)}{d} \sigma \mid u_R \mid u_{R,i} - \delta_{i3}g$$
 (4)

$$\frac{dx_{p,i}}{dt} = u_{p,i} \tag{5}$$

where t is time,  $u_{p,i}$  is the ith component of particle velocity,  $x_{p,i}$  is the particle position,  $C_d$  is drag coefficient (Morris 1976) which depends on particle Reynolds number,  $R_p$  ( $R_p = u_R d/\nu$ ),  $\nu$  is kinematic viscosity). The relative velocity of the particle, with respect to air,  $u_R$ , is given by

$$u_{R,i} = u_{p,i} - u_i \tag{6}$$

and

$$|u_R| = \sqrt{u_{R,1}^2 + u_{R,2}^2 + u_{R,3}^2}$$
 (7)

From the particle trajectories, we determine not only the speed, angle and location of particle impact, but also a number of other important physical quantities. These include streamwise sand flux density  $q_x$ ,  $q_y$  and  $q_z$ , and the particle momentum fluxes,  $au_{px}$ ,  $au_{py}$  and  $au_{pz}$ .

## 2.3 Aerodynamic entrainment

The motion of particles is initiated by aerodynamic shear stress. The aerodynamic ejection rate  $N_a$ , the number of particles entrained per unit area and per unit time (particle number flux density,  $[N_a]=[m^{-2}s^{-1})$  can be approximated by

$$N_a = \zeta_a(\tau_a - \tau_c) \tag{8}$$

where  $\zeta_a$  is the aerodynamic lift-off coefficient ( $[\zeta_a]=[N^{-1}s^{-1}]$ ),  $\tau_a=\rho u_*^2$  is shear stress at the surface and  $\tau_c=\rho u_{*t}^2$  is critical shear stress for entrainment,  $\rho$  is air density,  $u_*$  is friction velocity and  $u_{*t}$  is threshold friction velocity. Equation (8) is a physically plausible assumption. However, the magnitude of  $\zeta_a$  which is important for the model outcome, as shown later, deserves some consideration. It can be estimated that  $\zeta_a$  is around  $5\times 10^7~N^{-1}s^{-1}$ , for  $u_*=0.5~ms^{-1}$  for sand particles.

## 2.4 Splash entrainment

As saltating particles impact a mobile surface, they may rebound and eject more particles into the air. The splash scheme is a statistical description of the probability of rebound, the probability of ejection, and the velocity distributions of the rebound and ejected particles. In this model, we follow the basic notion of Anderson and Haff (1991) but take into account energetic constraints on rebound and splash.

The rebound probability is approximated by

$$P_r(v_I) = 0.95(1 - e^{-\gamma v_I}) \tag{9}$$

where  $v_I$  is particle impact velocity and  $\gamma$  is a dimensional parameter around 2 sm<sup>-1</sup>. If the total impact particle number is  $N_i$  and the impact velocity probability density function is  $p(v_I)$  then the total number of rebounds is

$$N_r = N_i \int_0^\infty P_r(v_I) p(v_I) dv_I \tag{10}$$

We assume an exponential rebounding velocity probability density function  $p(v_r, v_I)$ 

$$p_r(v_r, v_I) = \frac{1}{\alpha v_I} exp(-\frac{v_r}{\alpha v_I})$$
 (11)

where  $\alpha v_I$  is the mean velocity of  $v_r$  for given  $v_I$ , and  $\alpha$  is about 0.5 for typical saltation particles.

The splash rate is assumed to be proportional to impact velocity

$$n_s(v_I) = \zeta_s v_I \tag{12}$$

where  $\zeta_s$  is the splash coefficient. In principle,  $\zeta_s$  is assumed as a constant. If the total impact number is  $N_i$ , then the total number of splashed particles is

$$N_s = N_i \int_0^\infty \zeta_s v_I p(v_I) dv_I \tag{13}$$

Our chosen velocity probability density function of splashed particles is

$$p_s(v_s, v_I) = \frac{1}{hv_I^k} exp(-\frac{v_s}{hv_I^k})$$
(14)

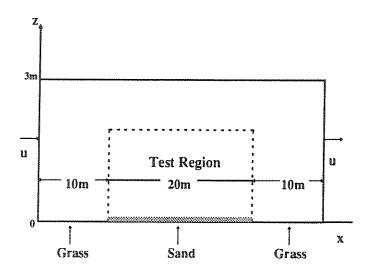


Figure 1: Configuration of numerical simulation.

where  $h = 0.25 \text{ m}^{0.7} \text{s}^{-0.7}$  and k = 0.3.

The kinetic energy available for rebound and splash is not independent. From this perspective, it can be shown that  $\zeta_s$  should obey

$$\zeta_s = \frac{v_I^{1-2k} \left(\frac{1-c_s}{2} - 0.95\alpha^2 \left(1 - e^{-\gamma v_I}\right)\right)}{h^2} \tag{15}$$

In general, the splash coefficient  $\zeta_s$  is not a constant, under the given assumptions of  $P_r$  and the velocity distribution functions for rebounding and splashing particles.

## 3 Results

The numerical model is intended to simulate the complex particle entrainment process in the atmospheric surface layer. In this paper, we concentrate on the so called Owen's phenomena. Owen's (1964) suggested that the atmospheric surface layer during saltation can be divided into an outer and inner layer. As far as the outer layer is concerned, the effect of saltation is an increase of surface roughness length. Owen (1964) and Chamberlain (1983) suggested that the saltation roughness length can be estimated by

$$z_{0S} = \frac{Cu_*^2}{2g} \tag{16}$$

where C is a constant of order 1. Raupach (1991) suggested that

$$z_{0S} = (Au_*^2/2g)^{1-\sqrt{r}} z_0^{\sqrt{r}}$$
(17)

where A is a constant,  $\sqrt{r}=u_{*t}/u_{*}$ ,  $z_{0}$  is aerodynamic roughness length when there is no saltation.

The configuration of the numerical experiment is illustrated in Figure 1. The domain of the simulation is 40m long and 3m high. The first 10m section and the last 10m section are nonerodible surfaces and the 20m middle section is mobile. The size of the saltating particles is assumed to be 250 micron, with a threshold friction velocity around 0.3m/s. The particle to air density ratio is 2200. The aerodynamic roughness length is assumed to be 1mm, and the upstream wind speed is assumed logarithmic.

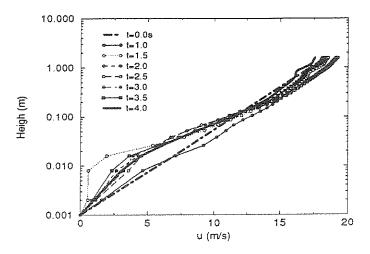


Figure 2: Simulated evolution of wind profile with saltation. The initial wind profile at time zero is logarithmic.

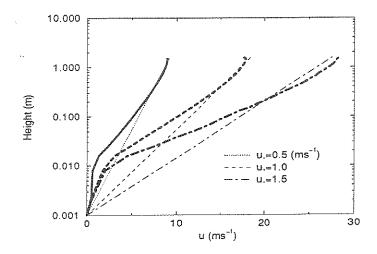


Figure 3: Surface wind profile modified by particle motion near the surface.

Figure 2 shows the evolution of wind profile at 15m downstream from the beginning of the erodible surface. Compared with the initial situation, the wind profile is significantly modified during particle saltation. Wind profile is rapidly adjusted to a new equilibrium within a few seconds. This can be seen from the fact that wind profile no longer changes substantially after 2s from the numerical simulation.

Wind speed at equilibration for three different  $u_*$  values are shown in Figure 3. The numerical results are qualitatively similar to those observed by Bagnold (1941). The saltating particles are accelerated by wind and extract momentum from the air. When these particles impact the surface, a proportion of their momentum is lost. This effect is to increase the surface roughness. Obviously, for a large surface friction velocity, more particles can be entrained into the atmosphere and accelerated to higher levels, leading to a larger saltation roughness length. The results shown in Figure 3 are as expected. The simulated results are substantially different from the analytical predictions of Raupach (1991). Both the numerical model and the analysis of Raupach needs to be verified using observed data.

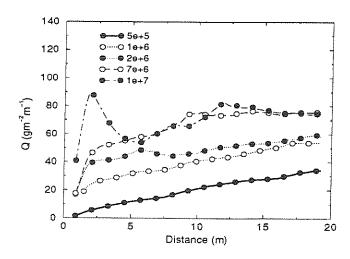


Figure 4: The sensitivity of streamwise sanddrift to the choice of aerodynamic liftoff coefficient  $\zeta_a$ .

The outcome of the numerical model depends on the choice of model parameters. Two parameters special to saltation are the aerodynamic liftoff coefficient  $\zeta_a$  and the splash coefficient  $\zeta_s$ . Figure 4 shows a sensitivity test of the streamwise sand drift to the choice of the aerodynamic liftoff coefficient,  $\zeta_a$ . The general behavior of streamwise sand drift is similar for the first four cases. For  $\zeta_a = 1e + 7$ , the streamwise sand flux shows an overshoot at a small distance downstream and equilibrate at about 15 meters downstream. At this stage of research, very little is know about the aerodynamic liftoff coefficient.

## References

Anderson, R. S. and Haff P. K., 1991: Wind Modification and Bed Response during Saltation of Sand in Air. *Acta Mechanica Suppl*, 1, 21-51.

Bagnold, R. A., 1941: The Physics of Blown Sand and Desert Dunes. Chapman and Hall, pp. 265. Chamberlain, A. C., 1983: Roughness length of sea, sand and snow. *Boundary-Layer Meteorol.*, 25, 405-409.

McEwan, I. K. and B. B. Willetts, 1991: Numerical Model of the Saltation Cloud. *Acta Mechanica*, *Suppl*, 1, 53-66.

Owen, R. P., 1964: Saltation of Uniform Grains in Air. J. Fluid Mech., 20, 225-242.

Raupach, M. R., 1991: Saltation Layers, Vegetation Canopies and Roughness Lengths. *Acta Mechanica Suppl*, 1, 83-96.

Shao, Y. and M. R. Raupach, 1992: The Overshoot and Equilibration of Saltation. J. Geophy. Res., 97, 20559-20564.

Shao, Y., M. R. Raupach and P. A. Findlater, 1993: Effect of Saltation Bombardment on the Entrainment of Dust by Wind. *J. Geophy. Res.*, 98, 12719-12726.